

Empirical Fit to Inelastic Electron-Deuteron and Electron-Neutron Resonance Region Transverse Cross Sections

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Abstract

An empirical fit is described to measurements of inclusive inelastic electron-deuteron cross sections in the kinematic range of four-momentum transfer $0 \leq Q^2 < 10 \text{ GeV}^2$ and final state invariant mass $1.1 < W < 3.2 \text{ GeV}$. The deuteron fit relies on a fit of the ratio R_p of longitudinal to transverse cross sections for the proton, and the assumption $R_p = R_n$. The underlying fit parameters describe the average cross section for a free proton and a free neutron, with a plane-wave impulse approximation used to fit to the deuteron data. Additional fit parameters are used to fill in the dip between the quasi-elastic peak and the $\Delta(1232)$ resonance. The mean deviation of data from the fit is 3%, with less than 4% of the data points deviating from the fit by more than 10%.

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I. INTRODUCTION

Empirical knowledge of the inclusive electron-deuteron cross section in the nucleon resonance region is important input for many research activities in nuclear and particle physics. The most important examples include calculations of radiative corrections to cross sections, extractions of spin structure functions from asymmetry measurements, determinations of structure function moments, and determinations of the vector coupling in models of low energy neutrino-nucleon cross sections. The latter is of particular importance since the quality of low energy neutrino-nucleon cross section models will become one of the largest uncertainties in the extraction of neutrino oscillation parameters from future long-baseline experiments.

In this paper we will describe a fit to precision electron-deuteron inelastic cross sections in the resonance region for negative four-momentum transfer $0 \leq Q^2 < 10 \text{ GeV}^2$ and final state invariant mass $1.1 < W < 3.2 \text{ GeV}$. Among the advantages over previous fits [1, 2] are: inclusion of photoproduction and low Q^2 data points; inclusion of several new experimental results; and the bulk of underlying fit is to the free nucleon (average of free proton and neutron), with Fermi motion consistently taken into account in a Plane Wave Impulse Approximation (PWIA). The latter provides for a parameter-free way of describing the broadening of resonance peaks with increasing three-momentum \vec{q} . The average nucleon transverse cross section fit form is the same as the recent fit to electron-proton data of Ref. [3], which uses a set of threshold-dependent Breit-Wigner forms for all resonances. The principal difference is that the present fit is to the transverse portion of the cross section only, due to a lack of sufficient virtual photon polarization ϵ range in the presently available deuteron data set. The assumption was made that the ratio of longitudinal to transverse cross sections, R , is the same for the free proton and neutron, as supported by a recent analysis of data with $W > 2 \text{ GeV}$ [4], and also supported by the limited study of the present analysis. With the above assumptions, the “dip” region between the quasi-elastic peak and the $\Delta(1232)$ resonance is under-predicted at low Q^2 , possibly because Meson Exchange Currents (MEC) and Final State Interactions (FSI) have been ignored [5]. An additional empirical function was used to fill in the missing strength in the “dip” region.

The overall structure of the fit can be summarized as:

$$\sigma_D^T(W, Q^2) = \sigma_{dip}(W, Q^2) + \int \sigma_N^T(W', (Q^2)') \Phi^2(\vec{k}) d^3\vec{k},$$

where $\sigma_D^T(W, Q^2)$ is the transverse cross section for a deuteron, $\sigma_{dip}(W, Q^2)$ is the dip region parametrization (6 free parameters), and the integral is over Fermi momentum \vec{k} of the average free nucleon transverse cross section $\sigma_N^T(W, Q^2)$ (36 free parameters). By simplifying the 3-dimensional integration to a 1-dimensional integration along the direction of the virtual photon, it was possible to simultaneously fit all 42 parameters in a single gradient search minimization. The minimization was done with respect to an ensemble of approximately 15,000 data points from 9 experiments.

The following section defines terms and kinematic variables. Section III describes the data sets used. Section IV gives details of the functional form used for $\sigma_{dip}(W, Q^2)$ and $\sigma_N^T(W, Q^2)$, and how the Fermi-smearing integral was simplified. The fit parameters are also listed in this section. In Section V, we discuss various features of the results.

II. DEFINITIONS AND KINEMATICS

In terms of the incident electron energy, E , the scattered electron energy, E' , and the scattering angle, θ , the absolute value of the exchanged 4-momentum squared in electron-nucleon scattering is given by

$$Q^2 = (-q)^2 = 4EE' \sin^2 \frac{\theta}{2}, \quad (1)$$

and the mass of the undetected hadronic system is

$$W^2 = M_p^2 + 2M_p\nu - Q^2, \quad (2)$$

with M_p the proton mass, $\nu = E - E'$, and the small terms involving the electron mass squared have been neglected.

In the one-photon exchange approximation, the spin-independent cross section for inclusive electron-nucleon scattering can be expressed in terms of the photon helicity coupling as

$$\frac{d\sigma}{d\Omega dE'} = \Gamma [\sigma_N^T(W, Q^2) + \epsilon \sigma_N^L(W, Q^2)], \quad (3)$$

where σ_N^T (σ_N^L) is the cross section for photo-absorption of purely transverse (longitudinal) polarized photons,

$$\Gamma = \frac{\alpha E' (W^2 - M_p^2)}{(2\pi)^2 Q^2 M_p E (1 - \epsilon)} \quad (4)$$

is the flux of transverse virtual photons, and

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1} \quad (5)$$

is the relative flux of longitudinal virtual photons. All the hadronic structure information is contained in σ_N^T and σ_N^L , which are only dependent on W and Q^2 . We use the definition $R_N = \sigma_N^L / \sigma_N^T$. We use the subscripts p , n , N , and D to refer to proton, neutron, average nucleon, and deuteron respectively, where the deuteron cross sections are defined to be per nucleon rather than per nucleus, following the high energy convention. We define $\sigma_N^{L,T} = (\sigma_p^{L,T} + \sigma_n^{L,T})/2$.

III. TREATMENT OF EXPERIMENTAL DATA USED IN FIT

A. Description of the Data Sets

The characteristics of the data sets used in the fit are summarized in Table I. The first reference (Ref. [6]) includes results from three early photoproduction experiments. We only used data from these experiments with beam energies above 800 MeV, because at lower energies the more recent photoproduction data of DAPHNE [7] has significantly smaller systematic errors. The JLab CLAS data [8] cover a wide kinematic range with many data points. The data are reported as values of the structure function F_2 , averaged over two different beam energies. Since the relative weight from the two beam energies (with differing values of ϵ) was not given, there is a systematic error in the conversion to σ_T that was taken into account in the total error bars. The early JLab Hall C data of Niculescu [2] cover a similar kinematic range as the CLAS data, with less data points but smaller statistical and systematic errors. The more recent JLab Hall C data of E00-116 [9] cover the high Q^2 range with relatively few data points, which nonetheless have very small statistical and systematic errors (typically a few percent). To extend the low W region to even higher Q^2 , although with larger relative errors, we included the data of SLAC E133 [10]. To cover the lower Q^2 region, we included preliminary data from two JLab Hall C experiments: E02-109* [13] and

E00-002 [14]. These experiments cover a wide range of ϵ for each (W, Q^2) point with small statistical errors. Systematic errors were being finalized at the time this fit was done, so we used 2% as a conservative estimate in lieu of the ultimate errors for this experiment.

| Data Set | Q_{Min}^2 (GeV ²) | Q_{Max}^2 (GeV ²) | # Data Points |
|------------------------------|------------------------------------|------------------------------------|---------------|
| Photoproduction (1972) [6] | 0 | 0 | 242 |
| Photoproduction (DAPHNE) [7] | 0 | 0 | 57 |
| CLAS [8] | 0.35 | 5.9 | 11725 |
| Early JLab [2] | 0.50 | 4.2 | 600 |
| JLab E00-116 [9] | 3.6 | 7.5 | 288 |
| SLAC E133 [10] | 2.5 | 10.0 | 488 |
| JLab E02-109* [13] | 0.02 | 2.0 | 1435 |
| JLab E00-002* [14] | 0.05 | 1.5 | 1445 |
| SLAC E140 [15] | 2.5 | 10.0 | 48 |

TABLE I: Data sets used in fit. The number of data points and the Q^2 range are indicated for each data set. *The data from Refs. [13, 14] are preliminary.

In order to constrain the fit at high W and Q^2 , where there are insufficient data from JLab, we included the DIS data from SLAC E140 [15], and also added pseudo-data points from the SMC [11] fit to DIS ed structure functions. The pseudo-data were generated over the interval $2.4 < W < 3.2$ GeV and $1.1 < Q^2 < 10$ GeV². The errors used were those given by the SMC fit.

The fit was found to be stable against the removal of any particular data set.

B. Quasi-elastic subtraction

Inelastic electron scattering on the deuteron can be divided into two distinct contributions: quasi-elastic scattering (just proton and neutron in the final state), and inelastic scattering (one or more mesons in the final state). Since the goal of the present work is to fit inelastic scattering on the average free nucleon, we have subtracted the quasi-elastic contribution (if not already done by the experimenters) using the model described in Appendix

I. Data points for which the quasi-elastic fraction was greater than 30% of the total cross section were discarded, as well as all points for which $W < 1.1$ GeV.

C. Longitudinal cross section subtraction

The next step in the data treatment was to extract σ_D^T from each of the electroproduction cross section measurements. This was done using:

$$\sigma_D^T = \sigma_D / (1 + \epsilon R_D).$$

As outlined in the introduction, we made the assumption that $R_n = R_p$, and evaluated R_D by Fermi-smearing both σ_p^L and σ_p^T from the proton fit of Ref. [3]. The Fermi-smearing procedure is described below. In practice, the Fermi-smearing had a very small effect for most (W, Q^2) points, so that, to a good approximation, $R_D = R_p$.

IV. DESCRIPTION OF THE FIT

A. Overview

A gradient-search minimization code (MINUIT) was used to simultaneously determine all fit parameters by minimizing the value of χ^2 , defined by:

$$\chi^2 = \sum_{i=1}^N [\sigma_i(W_i, Q_i^2) - \sigma_D^T(W, Q^2)]^2 / [\delta\sigma_i(W_i, Q_i^2)]^2$$

where the sum is over all experimental points with transverse inelastic cross section $\sigma_i(W_i, Q_i^2)$ and total statistical and systematic error $\delta\sigma_i(W_i, Q_i^2)$. To avoid a tedious iterative fit procedure, the Fermi-smearing integral in the model cross section $\sigma_D^T(W, Q^2)$ was simplified to a single-dimension integral, as explained below. Starting values of the average nucleon cross section $\sigma_N^T(W, Q^2)$ parameters were chosen to be the same as for the equivalent parameters in the proton fit of Ref. [3]. Some adjustments of the upper and lower limits on the parameters were needed to obtain the best χ^2 . The functional form and starting parameters for the “dip” cross section $\sigma_{dip}(W, Q^2)$ were obtained from detailed study of the fit residuals with the “dip” model absent.

In the next sections, we first describe the simplified Fermi-smearing procedure, then the functional form of the free average nucleon cross section, along with the fit parameters obtained, and finish with the fit form and parameters for the “dip” region.

B. Fermi-smearing

The Fermi-motion of the nucleons in the deuteron was taken into account using a PWIA calculation and the Paris [12] deuteron wave function $\Phi^2(\vec{k})$:

$$\sigma_D(W, Q^2) = \int \sigma_N(W', (Q^2)') \Phi^2(\vec{k}) d^3\vec{k} \quad (6)$$

where we made the approximations:

$$(W')^2 = \left(M_d + \nu - \sqrt{M^2 + \vec{k}^2} \right)^2 - (\vec{q})^2 - (\vec{k})^2 + 2\vec{q} \cdot \vec{k} \quad (7)$$

and $(Q^2)' = Q^2$, where M_d is the deuteron mass, M is the average nucleon mass, and ν and \vec{q} are the virtual photon energy and momentum, respectively. Since these equations basically boils down to the probability of finding a nucleon with longitudinal momentum k_z , with the z axis chosen along \vec{q} , we simplified the problem by determining 20 values of k_z^i for which the integral over $\Phi^2(\vec{k})$ is close to 1/20, in the special case $\sigma_N(W', (Q^2)') = 1$. The corresponding average values of $(k^2)_i$ were also evaluated. The cross section is then

$$\sigma_D(W, Q^2) = \sum_{i=1}^{20} \sigma_N(W'_i, (Q^2)') / 20 \quad (8)$$

where now

$$(W'_i)^2 = \left(M_d + \nu - \sqrt{M^2 + (k^2)_i} \right)^2 - (\vec{q})^2 - (k^2)_i + 2qk_z^i, \quad (9)$$

This simplification is made possible by assuming that there are no off-shell cross section corrections, and that the \vec{k}^2 terms are small enough that the integral over \vec{k}^2 at fixed k_z can be replaced by evaluating W' at the average values of \vec{k}^2 . This approximation works well for ranges in W over which the cross section is slowly varying. This is the case for the kinematic region of the present fit, but is not advisable for smearing into regions where the elementary cross section is zero (i.e. $W < M + M_\pi$). Hence, our fit is only valid for $W > 1.1$ GeV.

The numerical values of k_z^i and $(k^2)_i$ are listed in Table II.

Due to the rapid variation of the cross section near threshold, we used 200 bins instead of 20 for $W < 1.3$ GeV.

| $i =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| k_z^i | 0.0029 | 0.0083 | 0.0139 | 0.0199 | 0.0268 | 0.0349 | 0.0453 | 0.0598 | 0.0844 | 0.1853 |
| $(k^2)_i$ | 0.0050 | 0.0051 | 0.0055 | 0.0060 | 0.0069 | 0.0081 | 0.0102 | 0.0140 | 0.0225 | 0.0964 |

TABLE II: The first ten values of k_z^i and $(k^2)_i$, in units of GeV and GeV² respectively. The other 10 values are given by $k_z^{i+10} = -k_z^i$ and $(k^2)_{i+10} = (k^2)_i$.

C. Free Nucleon Fit Form

The fit form used to describe σ_N^T (the transverse cross section for the average of a proton and a neutron) used the same functional form as Ref. [3]. The total cross section is defined to be the incoherent sum of contributions from resonance production (σ^R) and a non-resonant background (σ^{NR}). The resonant cross section are described by threshold-dependent relativistic Breit-Wigner shapes with Q^2 -dependent amplitudes for each resonance, such that

$$\sigma^R(W^2, Q^2) = \sum_i BW^i(W^2) \cdot A_i^2(Q^2). \quad (10)$$

The form used for the Breit-Wigner resonance shapes is given by

$$BW^i = \frac{W K_i K_i^{cm}}{K K^{cm}} \cdot \frac{\Gamma_i^{\text{tot}} \Gamma_i^\gamma}{\Gamma_i [(W^2 - M_i^2)^2 + (M_i \Gamma_i^{\text{tot}})^2]}, \quad (11)$$

with

$$K = (W^2 - M_p^2)/2M_p, \quad (12)$$

$$K^{cm} = (W^2 - M_p^2)/2W. \quad (13)$$

Here, K and K^{cm} represent the equivalent photon energies in the lab and center of mass (CM) frames, respectively, while K_i and K_i^{cm} represent the same quantities evaluated at the mass of the i^{th} resonance, M_i . Γ_i^{tot} is the full decay width defined by

$$\Gamma_i^{\text{tot}} = \sum_j \beta_j^i \Gamma_j^i, \quad (14)$$

with β_j^i the branching fraction to the j^{th} decay mode for the i^{th} resonance and Γ_j^i the partial width for this decay mode. The partial widths for single pion or eta decay were defined as

$$\Gamma_j^i = \Gamma_i \left[\frac{p_j^{cm}}{p_j^{cm}|_{M_i}} \right]^{2L+1} \cdot \left[\frac{(p_j^{cm})|_{M_i}^2 + X_i^2}{(p_j^{cm})^2 + X_i^2} \right]^L, \quad (15)$$

where the p_j^{cm} are meson momenta in the center of mass, L is the angular momentum of the resonance, and X_i is a damping parameter. For two-pion decays, we used:

$$\Gamma_j^i = \frac{W\Gamma_i}{M_i} \left[\frac{p_j^{cm}}{p_j^{cm}|_{M_i}} \right]^{2L+4} \cdot \left[\frac{(p_j^{cm})^2|_{M_i} + X_i^2}{(p_j^{cm})^2 + X_i^2} \right]^{L+2}, \quad (16)$$

The virtual-photon width was defined by:

$$\Gamma_i^\gamma = \Gamma_i \left[\frac{K^{cm}}{K^{cm}|_{M_i}} \right]^2 \cdot \left[\frac{(K^{cm}|_{M_i})^2 + X_i^2}{(K^{cm})^2 + X_i^2} \right]^2. \quad (17)$$

Since $BW^i(W^2)$ depends only on W^2 , it was evaluated in 1 MeV bins in W from pion threshold to 5 GeV and stored in a look-up table for future reference. This significantly reduced the time needed for χ^2 evaluation needed by the fitting code.

| i | M_i | Γ_i | L^i | X_0^i | $\beta_{1\pi}^i$ | $\beta_{2\pi}^i$ | β_η^i | $A_i(0)$ | c_1^i | c_2^i | c_3^i |
|-----|-------|------------|-------|---------|------------------|------------------|----------------|----------|---------|---------|---------|
| 1 | 1.230 | 0.136 | 1 | 0.145 | 1.00 | 0.00 | 0.00 | 8.122 | 5.19 | 3.29 | 1.870 |
| 2 | 1.530 | 0.220 | 0 | 0.215 | 0.50 | 0.00 | 0.50 | 6.110 | -34.64 | 900.00 | 1.717 |
| 3 | 1.506 | 0.083 | 2 | 0.215 | 0.65 | 0.35 | 0.00 | 0.043 | 191.50 | 0.22 | 2.119 |
| 4 | 1.698 | 0.096 | 3 | 0.215 | 0.65 | 0.35 | 0.00 | 2.088 | -0.30 | 0.20 | 0.001 |
| 5 | 1.665 | 0.109 | 0 | 0.215 | 0.40 | 0.60 | 0.00 | 0.023 | -0.46 | 0.24 | 1.204 |
| 6 | 1.433 | 0.379 | 1 | 0.215 | 0.65 | 0.35 | 0.00 | 0.023 | 541.90 | 0.22 | 2.168 |
| 7 | 1.934 | 0.380 | 3 | 0.215 | 0.60 | 0.40 | 0.00 | 3.319 | 0 | 0 | 2.0 |

TABLE III: Resonance parameters for states included in the fit. The branching ratios to single pion, double pion, and η are denoted by $\beta_{1\pi}$, $\beta_{2\pi}$, and β_η respectively. The assumed angular momentum is denoted by L^i . Units of cross section are μb and all masses, momenta, and energies are in units of GeV.

For the transition amplitudes the fit form utilized was

$$A_i(Q^2) = \frac{A_i(0)}{(1 + Q^2/0.91)^{c_3^i}} \cdot \left(1 + \frac{c_1^i Q^2}{(1 + c_2^i Q^2)} \right). \quad (18)$$

The parameters for all the resonances are listed in Table III. The variables $A_i(0)$, c_1^i , c_2^i , and c_3^i were free parameters in the fit, while all other parameters were fixed to those used in the proton fit of Ref. [3].

The non-resonant background was parametrized as

$$\sigma^{NR} = \sum_{i=1}^2 x' (C_1^i (\delta W)^{(2i+1)/2}) / (Q^2 + C_2^i)^{(C_3^i + C_4^i Q^2 + C_5^i Q^4)} \quad (19)$$

where $\delta W = W - M_p - M_\pi$, M_π is the pion mass, and $x' = 1 + (W^2 - (M_p + M_\pi)^2) / (Q^2 + C_6)$, and the fit parameter $C_6 = 0.05 \text{ GeV}^2$. The fit values for the other ten parameters are listed in Table IV.

| i | C_1^i | C_2^i | C_3^i | C_4^i | C_5^i |
|---|---------|---------|---------|---------|-----------|
| 1 | 226.6 | 0.0764 | 1.4570 | 0.1318 | -0.005596 |
| 2 | -75.3 | 0.1776 | 1.6360 | 0.1350 | 0.005883 |

TABLE IV: Non-resonant parameters as described in the text. Units of cross section are μb and all masses, momenta, and energies are in units of GeV.

D. Dip region parametrization

It was found that with the assumption that the PWIA “smearing” of free nucleon cross sections describes electron-deuteron scattering, there was always missing strength in the “dip” region between the quasi-elastic peak at $W = M_p$, and the low W side of the $\Delta(1232)$ resonance. This missing strength could be due to MEC and FSI in either quasi-elastic or inelastic scattering. Whatever the physical cause, we chose a purely empirical form that greatly improved the fit quality. This dip region additive correction term is given by:

$$F_1^{\text{dip}} = 1.964 \nu^{0.298} e^{[-(W-1.086)^2/0.00531]} / (1 + \max(0.3, Q^2)/1.265)^8 \quad (20)$$

The unit-less structure function F_1 is related to σ_T by

$$F_1 = (W^2 - M_p^2) \sigma_T / 8\pi^2 \alpha (\hbar c)^2 \quad (21)$$

where α is the fine structure constant.

V. FIT RESULTS

The results of the deuteron fit are shown at seven representative values of Q^2 as a function of W in Fig. 1, along with the 1- σ error band. In order to span a smaller range on the vertical

axis, we plot F_1 rather than σ_T . Note that the resonant structure, clearly visible at low Q^2 , is essentially gone for $Q^2 > 5 \text{ GeV}^2$, due to the increasing influence of Fermi broadening and non-resonant background contributions.

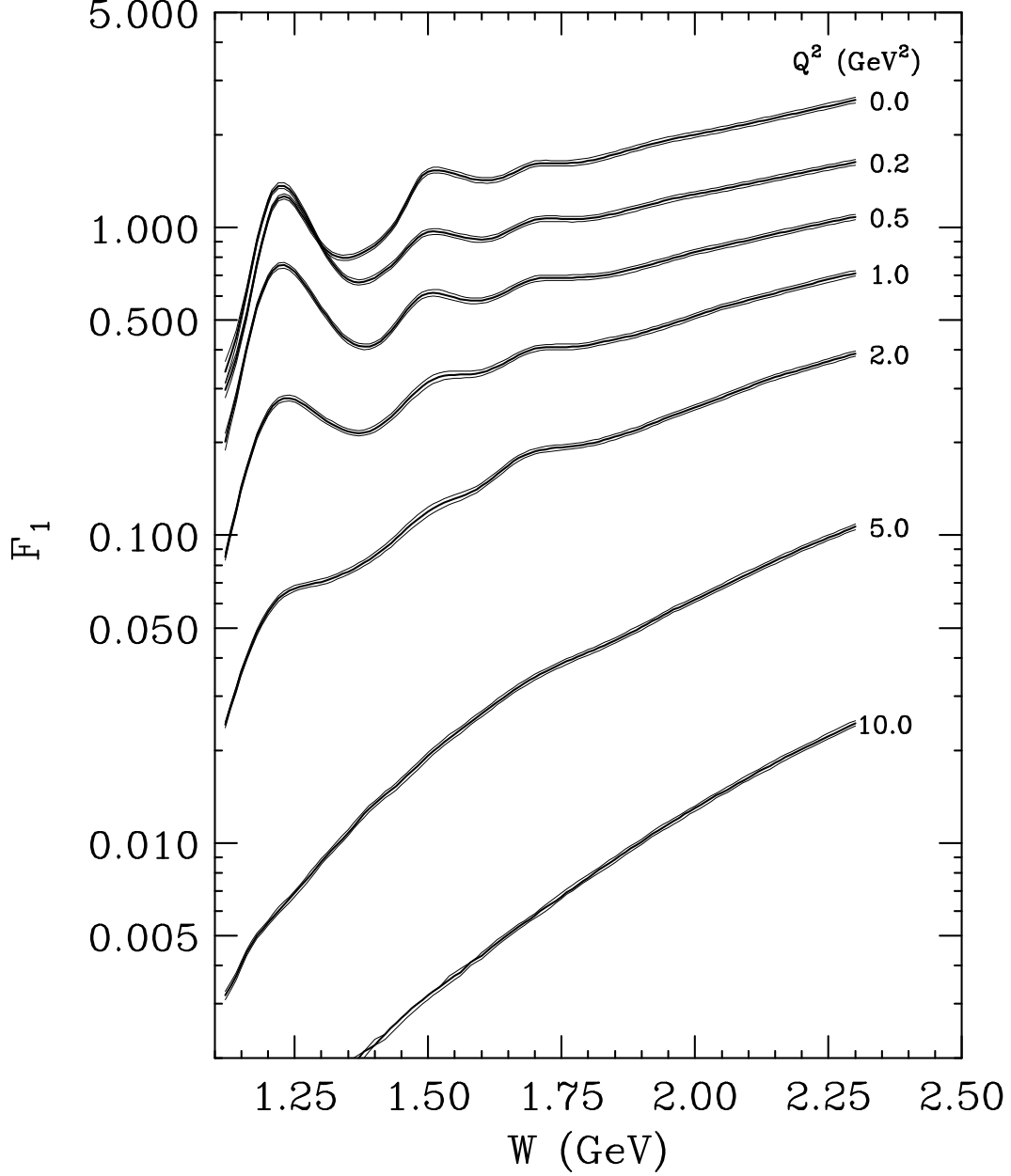


FIG. 1: Fit results for $F_1/\text{nucleon}$ for the deuteron as a function of W at representative values of Q^2 as indicated on the figure. Central values are shown as the thick lines, while the $1\text{-}\sigma$ error bands are shown as the thin lines.

Ratios of cross sections for each data point to the corresponding fit value are shown in Fig. 2 as a function of W for six bins in Q^2 . As shown in Fig. 3, 96% of the points lie

within 10% of the fit, 76% lie within 5%, and 53% lie within 3%. Overall, the agreement of data and fit is reasonably good at the 3% to 5% level. It can be seen in Fig. 2 that there are two noticeable oscillations in the ratios at low W and Q^2 : this trend is also seen in the proton fit of Ref. [3]. This may indicate the need for additional parameters to describe the photoproduction and very low Q^2 data.

In order to check the assumption that $R_p = R_n$, the data with $\epsilon < 0.5$ are plotted in gray (blue online), while those with $\epsilon > 0.5$ are plotted in black. No glaring discrepancy is seen between these two data sets. A more refined analysis will be performed once the preliminary data of JLab E02-109 [13] and E00-002 [14] are finalized, and results from the very recent JLab E06-009 [16] experiment become available. The E02-109 and E06-009 experiments were specifically designed to measure R_d , and cover a large range of ϵ at many specific values of (W, Q^2) .

The ratio of the commonly-used fit of Niculescu [2] to the present fit are also shown in Fig. 2. That fit tends to systematically lie above the data in the resonance region ($W < 2$ GeV), and lies well below data at high W and low Q^2 (outside the kinematic range of that fit).

Since the present underlying fit is to the average free nucleon, we can use the proton fit [3] to obtain predictions for the ratio of neutron to proton transverse cross sections (or equivalently the ratio F_1^n/F_1^p), as illustrated in Fig.4a. Significant resonance structure is predicted, especially at low Q^2 and in the region of the $\Delta(1232)$ resonance, for which the resonant contribution to F_1^n/F_1^p is expected to be unity by isospin invariance. These predictions can be tested against the anticipated results of the JLab “BONUS” experiment [17], which used tagging of low energy backward protons to “tag” spectator protons in electron-deuteron scattering (and hence isolate electron-neutron scattering). Predicted ratios of F_1^n/F_1^d that could be extracted from BONUS are shown in Fig.4b at three representative values of Q^2 .

VI. SUMMARY

An empirical fit to inelastic electron-deuteron scattering has been performed which describes available data reasonably well (3% to 5% level) in nearly all of the kinematic range than can be accessed at Jefferson Lab with up to 6 GeV electrons and photons: $0 \leq Q^2 < 10$ GeV² and $1.1 < W < 3.2$ GeV. The fit is useful in the evaluation of radiative corrections

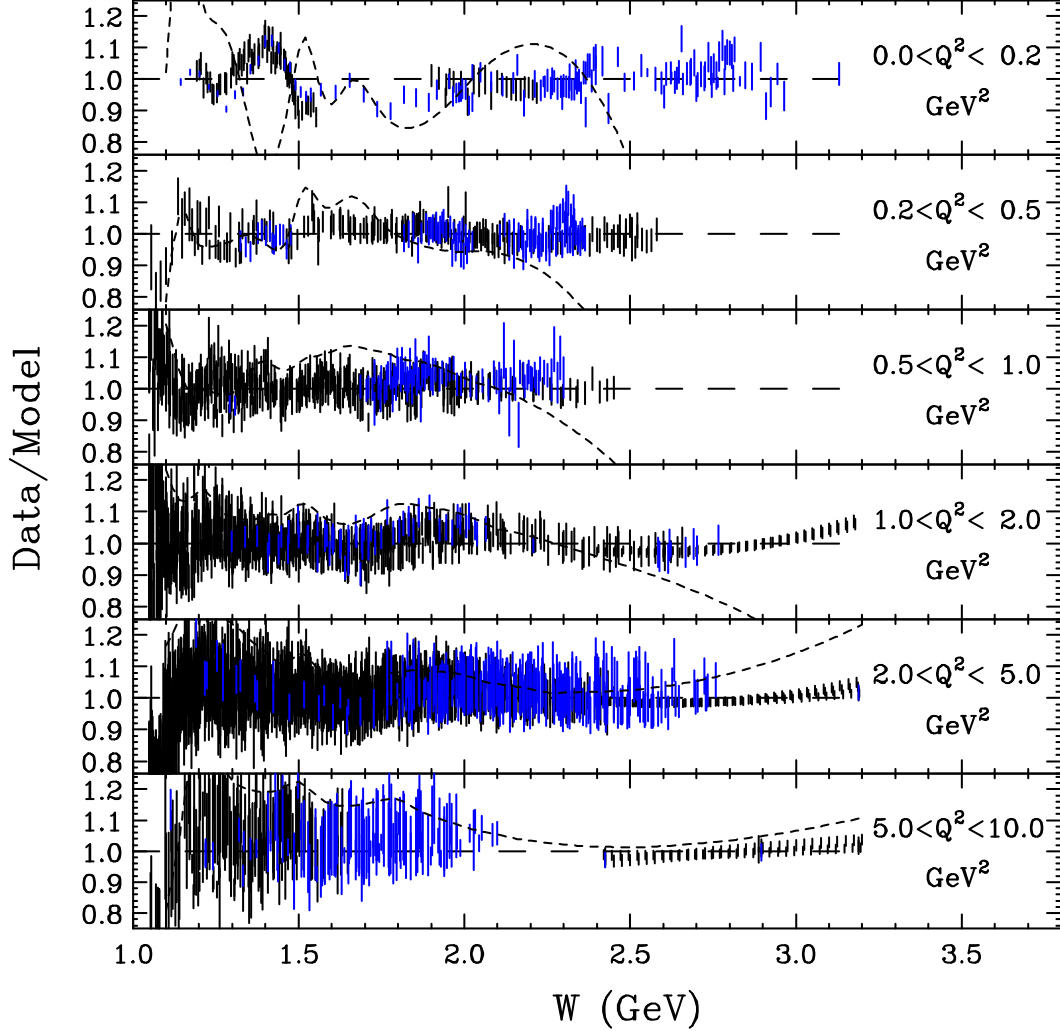


FIG. 2: Ratio of ed inelastic data to model as a function of W in six ranges of Q^2 . The gray (blue online) points correspond to $\epsilon < 0.5$, while the black points are for $\epsilon > 0.5$. Most of the points for $W > 2.4$ GeV and $Q^2 > 1$ GeV² are from the ed fit from SMC [11]. The ratio of the fit of Niculescu [2] to the present fit is illustrated by the dashed curves.

to experimental data, for extraction of spin structure functions from asymmetry measurements, and for the evaluation of structure function moments. Since the underlying fit is to an average nucleon, the results can be combined with a proton fit [3] to obtain predictions for electron-neutron scattering in the resonance region. Suitably corrected for Fermi motion, these can in turn be used to make neutron excess corrections to nuclear structure functions.

Once the data from JLab E02-109 [13], E00-002 [14], and JLab E06-009 [16] are finalized, we plan to re-do the fit for σ_L and σ_T separately, rather than making the assumption

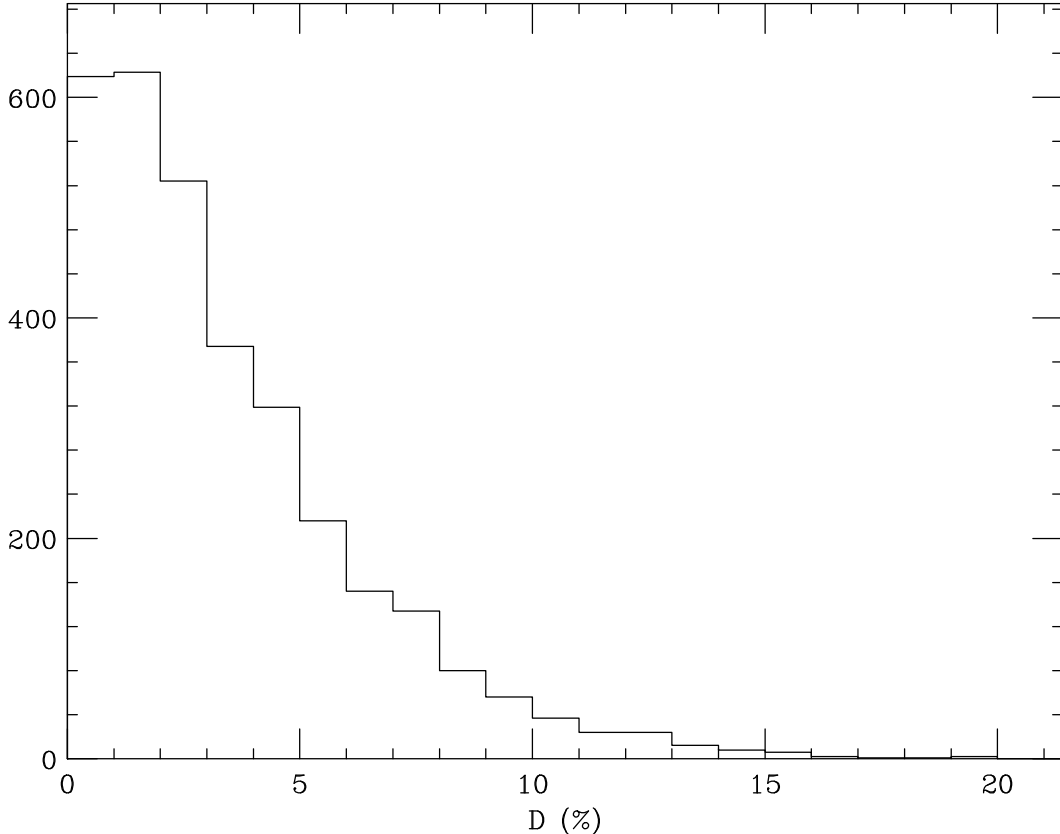


FIG. 3: Frequency distribution for the deviations from unity of the ratios of data to fit.

$R_p = R_n$. We also plan to use the results of the BONUS [17] experiment, which to first order will measure the ratio of electron-neutron to electron-deuteron scattering.

FORTTRAN computer code embodying the electron-deuteron fit described in this article is available by email request from the authors. The code includes the fit covariance matrix (or error matrix), and a subroutine to obtain the error on F_1^d . The code also includes the proton fit of Ref. [3], permitting the determination of electron-neutron cross sections from the deuteron and proton fits. The same code also includes a simple Fermi-smearing model of electron-nucleus cross sections for $A > 2$, using these deuteron and proton fits, as described in Ref. [21]. The quasi-elastic model described in the Appendix is also included.

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VII. APPENDIX I: QUASI-ELASTIC MODEL

To model quasi-elastic scattering, we used the same PWIA Fermi-smearing prescription (based on the deuteron Paris wave function) as for inelastic scattering, except that the continuous inelastic cross section was replaced by a δ -function elastic cross at $W = M_p$. The elastic cross section was calculated using the nucleon form factors of Bosted [18], modified for off-shell effects using the prescription of Ref. [19]. Following Tsai [20], Pauli suppression was taken into account using the factor $(3q/4k_f)1 - [(q/k_f)^2]/12$, for $q < 2k_f$, and unity for $q > 2k_f$, where q is the magnitude of \vec{q} , and we used $k_f = 0.085$ GeV. As shown in Fig. 5, the model works reasonably well near the quasi-elastic peak at two values of Q^2 . It can also be seen that the empirical “dip region” parametrization is useful to describe the region near $W = 1.09$ GeV at low Q^2 .

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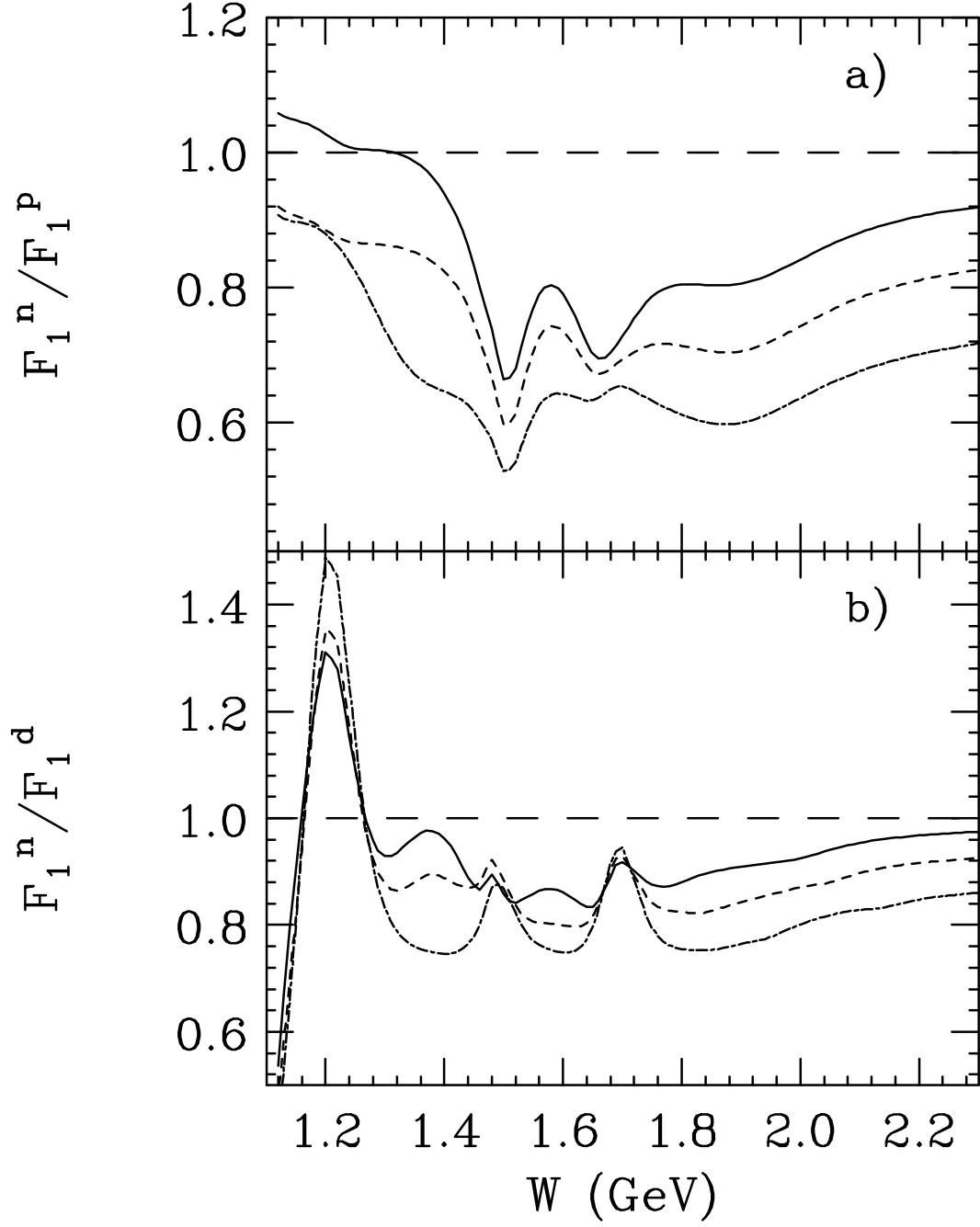


FIG. 4: Fit results for a) the ratio F_1^n/F_1^p and b) the ratio F_1^n/F_1^d as a function of W for $Q^2 = 0.5$ GeV² (solid curves), $Q^2 = 1$ GeV² (dashed curves), and $Q^2 = 2$ GeV² (dot-dashed curves). F_1^d is defined to be per nucleon.

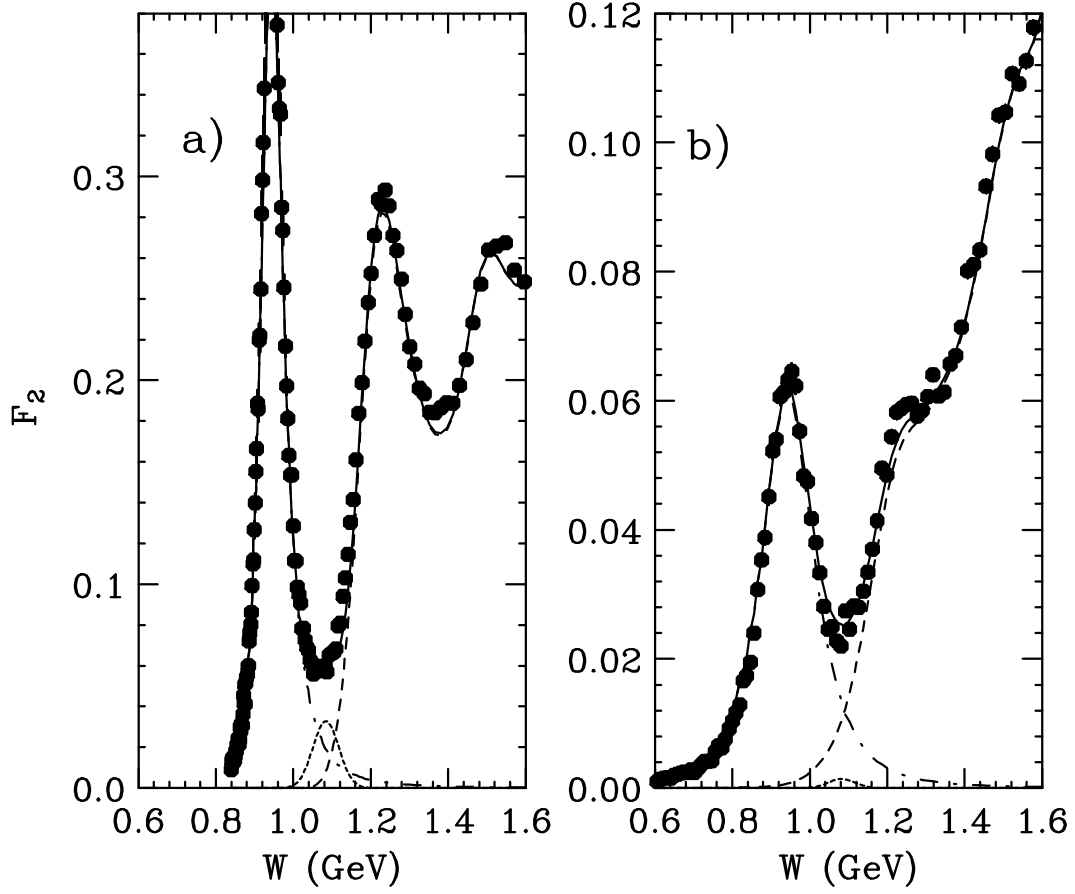


FIG. 5: Comparison of the quasi-elastic model (dot-dashed curves), PWIA part of the inelastic model (dashed curves), “dip region” part of the inelastic model (short dashed curves), and their sum (solid curves) with F_2 data from Ref. [8] at a) $Q^2 = 0.525 \text{ GeV}^2$ and b) $Q^2 = 2.075 \text{ GeV}^2$.